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Appendix A: Solving the wave equation in cylindrical coordinates

The solution of the wave equation expressed in cylindrical coordinates leads to modified Bessel equation, which is just the Bessel equation for an imaginary argument. This result is used in section 4.2.1.

Consider:

$$f_{:rr} + \frac{1}{r} f_{:r} - \frac{n^2}{r^2} f + f_{:zz} = \frac{1}{c^2} f_{:tt} \quad \textbf{(A-1)}$$

Seek separable solutions:

$$f(r,z,t) = R(r)Z(z)T(t) \quad \textbf{(A-2)}$$

This yields:

$$\frac{R_{:rr}}{R} + \frac{1}{r}\frac{R_{:r}}{r} - \frac{n^2}{r^2} + \frac{Z_{:zz}}{Z} = \frac{1}{c^2}\frac{T_{:tt}}{T} \quad \textbf{(A-3)}$$

Since *R(r)*, *Z(z)* and *T(t)* are independent functions it follows that:

$$l^2 - k_z^2 = -k^2 \quad \textbf{(A-4)}$$

where:

$$\frac{R_{:rr}}{R} + \frac{1}{r}\frac{R_{:r}}{r} - \frac{n^2}{r^2} = l^2 \quad \textbf{(A-5)}$$

$$\frac{Z_{:zz}}{Z} = -k_z^2 \quad \textbf{(A-6)}$$

$$\frac{1}{c^2}\frac{T_{:tt}}{T} = -k^2 \quad \textbf{(A-7)}$$

Equation A-6 has the trivial solution:

$$Z = e^{-ik_z z} \tag{A-8}$$

where the sign of the exponential has been chosen for convenience. Equation A-7 has a similar solution:

$$T = e^{ikct} = e^{i\omega t} \tag{A-9}$$

where again the sign of the exponential has been chosen for convenience, and:

$$\omega = kc \tag{A-10}$$

Now solve A-5. First rearrange:

$$r^2 R_{;rr} + rR_{;r} - (n^2 + l^2 r^2)R = 0 \tag{A-11}$$

This is just the modified Bessel equation which has the solution:

$$R(lr) = AK_n(lr) + BI_n(lr) \tag{A-12}$$

where I_n is the modified Bessel function of the first kind, of order *n* and K_n is the modified Bessel function of the second kind of order *n*. Hence a solution of A-2 is:

$$f(r,z,t) = \{ AK_n(lr) + BI_n(lr) \} e^{i(\omega t - k_z z)} \tag{A-13}$$

The general solution is formed by a sum over all combinations of k_z and ω which gives:

$$f(r,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \{ AK_n(lr) + BI_n(lr) \} e^{i(\omega t - k_z z)} dk_z d\omega \tag{A-14}$$

Note: This is just the double Fourier transform of $f(r,k_z,\omega,)$ where $f(r,k_z,\omega) = AK_n(lr) + BI_n(lr)$.

Equations A-4 and A-10 together imply:

$$l^2 = k_z^2 - \frac{\omega^2}{c^2} \quad \textbf{(A-15)}$$

Equations A-14 and A-15 give the general solution to equations of the form A-1.

The scalar wave equation for P waves in cylindrical co-ordinates is:

$$\phi_{:rr} + \frac{1}{r}\phi_{:r} + \phi_{:zz} = \frac{1}{\alpha^2}\phi_{:tt} \quad \textbf{(4-1)}$$

where the P wave displacement potential $\phi = \phi(r,z,t)$ and α is the P wave speed. Note this is of the same form as A-1, with n=0. Hence the solution is given by A-14:

$$\phi(r,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{AK_0(l_p r) + BI_0(l_p r)\right\}e^{i(\omega t - k_z z)}dk_z d\omega \quad \textbf{(A-16)}$$

with

$$l_p^2 = k_z^2 - \frac{\omega^2}{\alpha^2} \quad \textbf{(A-17)}$$

The scalar wave equation for SV waves in cylindrical coordinates is:

$$\psi_{:rr} + \frac{1}{r}\psi_{:r} - \frac{1}{r^2}\psi + \psi_{:zz} = \frac{1}{\beta^2}\psi_{:tt} \quad \textbf{(4-4)}$$

where $\psi(r,z,t)$ is the scalar SV displacement potential. This is just A-1 again, with f=ψ and n=1. Hence the solution is given by A-14 as:

$$\psi(r,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{AK_1(l_s r) + BI_1(l_s r)\right\}e^{i(\omega t - k_z z)}dk_z d\omega \quad \textbf{(A-18)}$$

with:

$$l_s^2 = k_z^2 - \frac{\omega^2}{\beta^2} \quad \textbf{(A-19)}$$

The above solution can be used to study waves that originate in the fluid and are refracted into the solid at the conduit wall. This solution includes body waves (P and

S waves) as well as interface waves such as Stoneley waves and pseudo-Rayleigh waves [*Biot*, 1952; *Cheng and Toksoz*, 1980].

Waves which are reflected at the conduit wall correspond to real, rather than imaginary, radial wavenumbers. These lead to the standard Bessel equation the solution to which involves standard Bessel functions (J_n and Y_n) rather than the modified Bessel functions used throughout this thesis.

Appendix B: Physical interpretation of solutions to the wave equation

The wave equation solutions in 4-10 and 4-11 can be thought of as describing the wavefield in terms of an infinite sum of plane waves propagating out from the origin at all speeds and at all angles. This is because the integrand itself is approximately the mathematical expression of a plane wave, which is true because the modified Bessel functions [used in Chapter 4] are closely related to the exponential function for small arguments. The superposition of these waves results in spherical waves. Consider:

$$BI_o(lr)e^{-ik_z z}e^{i\omega t} \qquad \textbf{(B-1)}$$

Replacing l by the radial wavenumber k_r gives:

$$BI_o(-ik_r r)e^{-ik_z z}e^{i\omega t} \qquad \textbf{(B-2)}$$

But for small arguments, $I_0(x) \sim e^x$ which leads to:

$$B\exp(-i(k_r r + k_z z - \omega t) \qquad \textbf{(B-3)}$$

which expresses a plane wave.

By summing over all frequencies for constant vertical wavenumber the radial wavenumber expressed in 4-7 and 4-8 changes, i.e. the direction of plane wave propagation changes. By summing over all frequencies and all vertical wavenumbers, the phase speed of the plane wave takes all values too. Phase speed is given by:

$$v = \pm\frac{\omega}{k_z} \qquad \textbf{(B-4)}$$

Appendix C: Derivatives of modified Bessel functions

The recurrence relations for the modified Bessel function of the first kind, $I_n(x)$, are:

$$I_n(x) = I_{-n}(x)$$

$$I_{n-1}(x) + I_{n+1}(x) = 2I_{n:x}(x)$$

$$I_{n-1}(x) - I_{n+1}(x) = \frac{2nI_n}{x}$$

These lead to useful formulae for the derivatives of $I_0(lr)$ and $I_1(lr)$:

$$\frac{\partial I_0(lr)}{\partial r} = I_{0:r} = lI_1 \tag{C-1}$$

$$\frac{\partial I_1(lr)}{\partial r} = I_{1:r} = (lI_0 - I_1 r^{-1}) \tag{C-2}$$

Similarly the recurrence relations for the modified Bessel function of the second kind, $K_n(x)$, are:

$$K_n(x) = K_{-n}(x)$$

$$K_{n-1}(x) + K_{n+1}(x) = -2K_{n:x}(x)$$

$$K_{n-1}(x) - K_{n+1}(x) = -\frac{2nK_n}{x}$$

These lead to useful formulae for the derivatives of $K_0(lr)$ and $K_1(lr)$:

$$\frac{\partial K_0(lr)}{\partial r} = K_{0:r} = -lK_1 \tag{C-3}$$

$$\frac{\partial K_1(lr)}{\partial r} = K_{1:r} = -(lK_0 + K_1 r^{-1}) \tag{C-4}$$

Appendix D: Modified Bessel functions and the singularities of $K_n(lr)$

The modified Bessel functions $I_n(x)$ and $K_n(x)$ are equivalent to the usual Bessel functions $J_n(x)$ and $Y_n(x)$ evaluated for purely imaginary arguments.

$$I_n(x) = (-i)^n J_n(ix)$$
$$K_n(x) = \frac{\pi}{2} i^{n+1}\left[J_n(ix) + iY_n(ix)\right]$$

This choice makes the functions real valued for real arguments *x*. The graph of these functions shows that $K_n(x)$ has a singularity for $x=0$, and that $I_n(x)$ increases approximately exponentially.

For $x>>n$ the following formulae are valid [*Press et al.*, 1992, p229-231]:

$$I_n(x) \approx \frac{1}{\sqrt{2\pi x}} e^x$$
$$K_n(x) \approx \frac{\pi}{\sqrt{2\pi x}} e^{-x}$$

Singularities of $K_n(lr)$ correspond to $lr=0$, that is:

$$K_n(0) \equiv \infty \qquad \textbf{(D-1)}$$

Hence there are singularities at $r=0$ and $l=0$. The singularity at $r=0$ means that $K_n(lr)$ cannot describe the wavefield in the fluid unless a source is present. More interesting is the singularity at $l=0$. This implies:

$$l^2 = k_z^2 - \frac{\omega^2}{c^2} = 0 \qquad \textbf{(D-2)}$$

Or:

$$k_z = \pm \frac{\omega}{c} \qquad \textbf{(D-3)}$$

So singularities occur for real values of k_z. But if a (constant) imaginary part is introduced to the frequency ω then singularities only occur for complex values of k_z, and it becomes possible to integrate along the real k_z axis.

Appendix E: Fourier transforms

Throughout this thesis the conventions adopted for Fourier transforms are:

$$f(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \tag{E-1}$$

$$f(k_z) = \int_{-\infty}^{\infty} f(z) e^{ik_z z} dz \tag{E-2}$$

which imply the corresponding inverse Fourier transforms:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) e^{i\omega t} d\omega \tag{E-3}$$

$$f(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(k_z) e^{-ik_z z} dk_z \tag{E-4}$$

We use the following special functions:

$$\delta(x - x_0) = \begin{cases} 1 & x = x_0 \\ 0 & x \neq x_0 \end{cases}$$ the Dirac delta function

$$H(x - x_0) = \int_{-\infty}^{\infty} \delta(x - x_0) dx = \begin{cases} 1 & x > x_0 \\ 0 & x < x_0 \end{cases}$$ the Heaviside function.

$$B(x)_{x_0}^{x_1} = H(x - x_1) - H(x - x_0) = \begin{cases} 0 & x > x_1 \\ 1 & x_0 < x < x_1 \\ 0 & x < x_0 \end{cases}$$ the Boxcar function.

Some useful transforms pairs are:

$$\delta(t - t_0) \Leftrightarrow e^{-i\omega t_0} \tag{E-5}$$

$$H(t-t_0) \Leftrightarrow \frac{e^{-i\omega t_0}}{i\omega} \tag{E-6}$$

$$B(t)_{t_0}^{t_1} \Leftrightarrow \frac{e^{-i\omega t_0} - e^{-i\omega t_1}}{i\omega} \tag{E-7}$$

$$e^{i\omega_0 t} B(t)_{t_0}^{t_1} \Leftrightarrow \frac{e^{-i(\omega-\omega_0)t_0} - e^{-i(\omega-\omega_0)t_1}}{i(\omega-\omega_0)} \tag{E-8}$$

$$tB(t)_{t_0}^{t_1} \Leftrightarrow \frac{e^{-i\omega t_1}}{\omega^2}(1+i\omega t_1) - \frac{e^{-i\omega t_0}}{\omega^2}(1+i\omega t_0) \tag{E-9}$$

and:

$$\delta(z-z_0) \Leftrightarrow e^{ik_z z_0} \tag{E-10}$$

$$H(z-z_0) \Leftrightarrow -\frac{e^{-ik_z z_0}}{ik_z} \tag{E-11}$$

$$B(z)_{z_0}^{z_1} \Leftrightarrow \frac{e^{ik_z z_1} - e^{ik_z z_0}}{ik_z} \tag{E-12}$$

$$e^{-ik_0 z} B(t)_{z_0}^{z_1} \Leftrightarrow \frac{e^{i(k_z-k_0)z_1} - e^{i(k_z-k_0)z_0}}{i(k_z-k_0)} \tag{E-13}$$

$$zB(z)_{z_0}^{z_1} \Leftrightarrow \frac{e^{ik_z z_1}}{k_z^{\,2}}(1-ik_z z_1) - \frac{e^{-ik_z z_0}}{k_z^{\,2}}(1-ik_z z_0) \tag{E-14}$$

These results are used extensively in section 4-4 and listed here because they are not included in advanced mathematical textbooks.